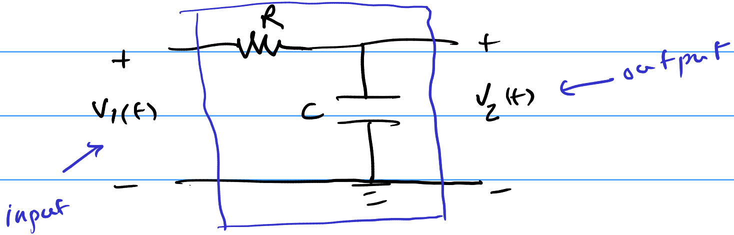
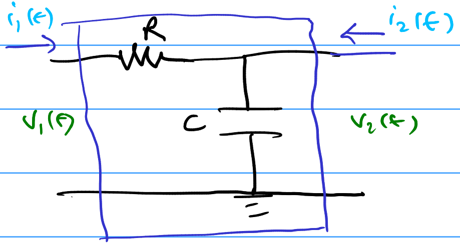
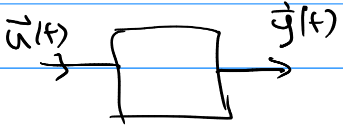


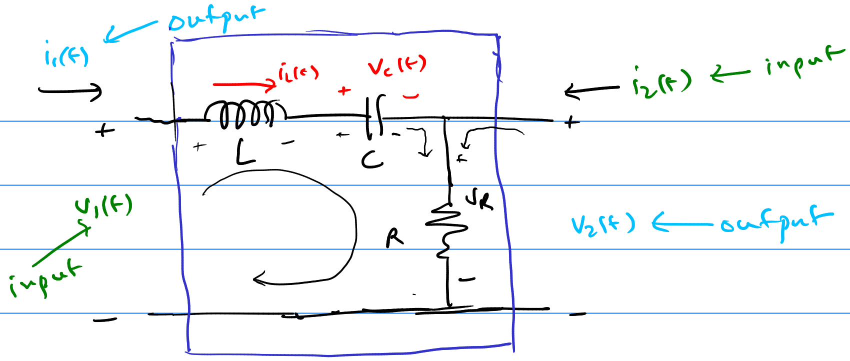


inputs

$$\vec{u}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} \qquad \vec{y}(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$



$$V_R = R(i_2 + i_L)$$



$$\vec{u}(t) = \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

$$\vec{x}(t) = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$$

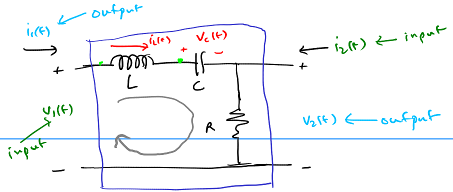
STATE

$v_1(t)$

<u>KCL</u>: $i_L = C \dfrac{dv_c}{dt}$

<u>KVL</u>: $v_1(t) = L \dfrac{di_L}{dt} + v_c(t) + \overbrace{R(i_2(t) + i_L(t))}^{V_R}$

$$\vec{y}(t) = \begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}$$



KCL: $i_L = C\frac{dv_c}{dt}$

KVL: $v_1(t) = L\frac{di_L}{dt} + v_c(t) + R(i_2(t) + i_L(t))$

$$\rightarrow \frac{dv_c}{dt} = \frac{i_L}{C}$$

$$\rightarrow \frac{di_L}{dt} = \frac{v_1(t) - v_c(t) - R(i_2(t) + i_L(t))}{L}$$

$\vec{u}(t) = \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$

$\vec{x}(t) = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$

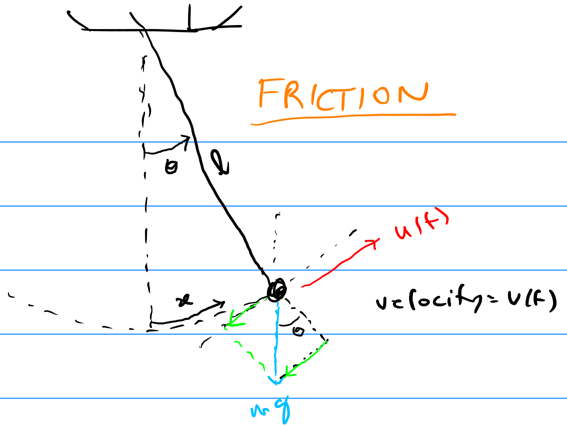
STATE

$\vec{y}(t) = \begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}$

$$\frac{d}{dt}\overbrace{\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}}^{\vec{x}} = \begin{bmatrix} 0 & 1/C \\ -\frac{1}{L} & -R/L \end{bmatrix} \overbrace{\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}}^{\vec{x}} + \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} \overbrace{\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}}^{\vec{u}(t)}$$

STATE SPACE FORMULATION

PENDULUM



→ tangential force on bob: $-mg\sin(\theta)$

→ input force: $u(t)$

→ friction: $-k\,v(t) = -k\ell\, v_\theta(t)$

→ $x(t)$ = arc-length from vertical (+ve = right)

→ $v(t) = \frac{dx}{dt}$, arc = $a = \frac{dv}{dt} = \frac{dx^2}{dt^2}$; $x = \ell\theta$ ← radians

→ Newton's Law: force = $m\,a$

→ tangential force on bob: $-mg\sin(\theta)$

→ input force: $u(t)$

→ friction: $-k\,v(t)$

→ $x(t)$ = arc-length from vertical (+ve = right)

→ $v(t) = \frac{dx}{dt}$, acc = $a = \frac{dv}{dt} = \frac{dx^2}{dt^2}$;

→ Newton's Law: force = m a

$$x(t) = l\,\theta(t)$$

$$v(t) = l\,\frac{d\theta}{dt} \;; \qquad \frac{d\theta}{dt} \to v_\theta(t)$$

$$a(t) = l\,\frac{d^2\theta}{dt} = l\,\frac{d\,v_\theta(t)}{dt}$$

total Force (towards the right) = $-mg\sin(\theta) - kv(t) + u(t)$

equ. 1 →

$$\boxed{\begin{aligned} \frac{dv_\theta}{dt} &= -\frac{\cancel{m}g}{l}\sin(\theta) - \frac{kv_\theta}{m} + \frac{u(t)}{ml} \\ \frac{d\theta}{dt} &= v_\theta \end{aligned}}$$

$$\vec{x} = \begin{bmatrix} \theta \\ v_\theta \end{bmatrix} \qquad \frac{d\vec{x}}{dt} = \vec{f}(\vec{x}, u(t))$$

DISCRETE TIME SYSTEMS

r = yearly interest

P = principal

0, 1, 2, 3, 4, 5

additional money put in / taken out

$$\boxed{S(t+1) = S(t) + S(t)\frac{r}{12} + u(t)}$$

current month number

$$\underbrace{S(t+1) - S(t)} = S(t)\frac{r}{12} + u(t)$$

$p(t)$: profs
$r(t)$: PhDs
β: fraction of PhDs becoming profs
δ: " who "leave" the profession every year
$u(t)$: # of PhDs produced per prof.

$$p(t+1) = p(t) + \beta r(t) - \delta p(t)$$

$$r(t+1) = r(t) - \beta r(t) - \delta r(t) + u(t)\, p(t)$$

$$\vec{x}(t) = \begin{bmatrix} p(t) \\ r(t) \end{bmatrix} \qquad \begin{bmatrix} p(t+1) \\ r(t+1) \end{bmatrix} = \underbrace{\begin{bmatrix} p(t) + \beta r(t) - \delta p(t) \\ r(t) - \beta r(t) - \delta r(t) + u(t)\, p(t) \end{bmatrix}}_{\vec{f}(\vec{x}(t),\, u(t))}$$

$$\frac{dx}{dt} = f(x) + u(t)$$

$$x \rightarrow x^* + \underbrace{\Delta x(t)}_{\text{small}}$$

$$u(t) \rightarrow u^* + \underbrace{\Delta u(t)}_{\text{small}}$$

$$\frac{d}{dt}\left(x^* + \Delta x(t)\right) = f\left(x^* + \Delta x(t)\right) + u^* + \Delta u(t)$$

T.S.

$$\frac{d}{dt}\Delta x(t) = \cancel{f(x^*)} + \left.\frac{df}{dx}\right|_{x^*} \Delta x(t) + \cancel{u^*} + \Delta u(t)$$

$$\left. df/dx \right|_{x^*} \qquad \frac{df}{dx}(x^*) \qquad f'(x^*)$$

$$\boxed{\frac{d}{dt}\Delta x(t) = J\,\Delta x(t) + \Delta u(t)}$$

$$\vec{x} = \begin{bmatrix} \theta \\ v_\theta \end{bmatrix}$$

DC op' pt.

$u^* = 0 \Rightarrow x^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is a DC op pt

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} v_\theta \\ -g/\ell \sin(\theta) - k/m\, v_\theta + \frac{b(t)}{m\ell} \end{bmatrix}$$

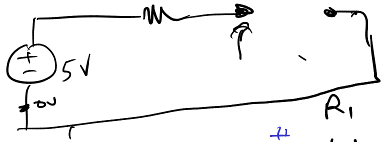
$\frac{\partial}{\partial\theta}$

$\vec{x} = \begin{bmatrix} \theta \\ v_\theta \end{bmatrix}, \quad \vec{u} = [b(t)]$

$$J_x = \begin{bmatrix} 0 & 1 \\ -g/\ell \cos(\theta) & -k/m \end{bmatrix}$$

x^*, u^*

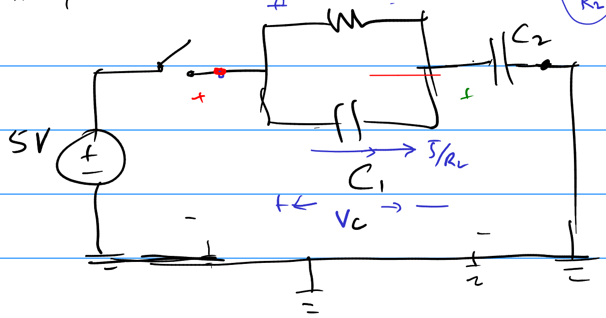
$$\begin{bmatrix} 0 & 1 \\ -g/\ell & -k/m \end{bmatrix}$$

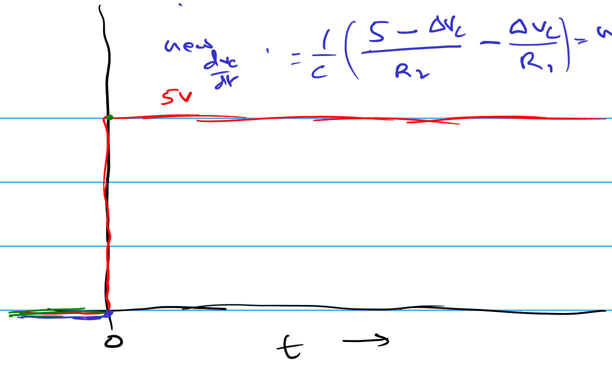


$V_c : 0 \longrightarrow \Delta V_c$ in dt

$\frac{5 - V_c(t)}{R_2}$

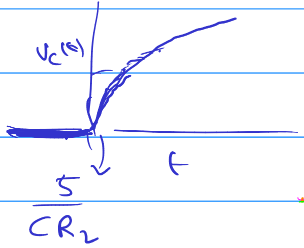
new $\frac{dv_c}{dt} = \frac{1}{C}\left(\frac{5 - \Delta V_c}{R_2} - \frac{\Delta V_c}{R_1}\right)$ = new current

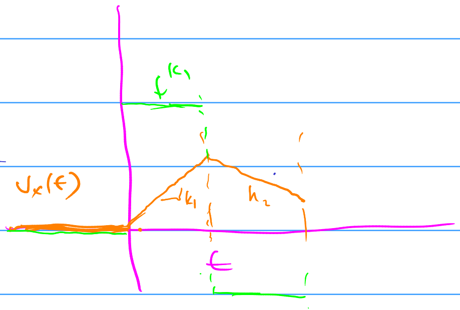




$i_c = \frac{dv_c}{dt}$

$\frac{5}{R_2 C} = \frac{dv_c}{dt}$





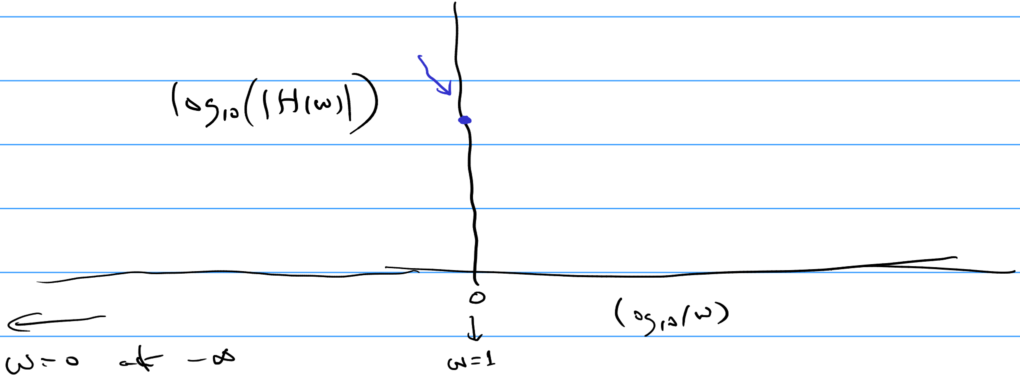
voltage xfer fn.

$$H(\omega) = \frac{1}{j\omega + a} \implies |H(\omega)| = \frac{1}{|j\omega + a|} = \frac{1}{\sqrt{\omega^2 + a^2}} = (\omega^2 + a^2)^{-\frac{1}{2}}$$

real number

real

$$20 \log_{10}(|H(\omega)|) = -10 \log_{10}(\omega^2 + a^2)$$



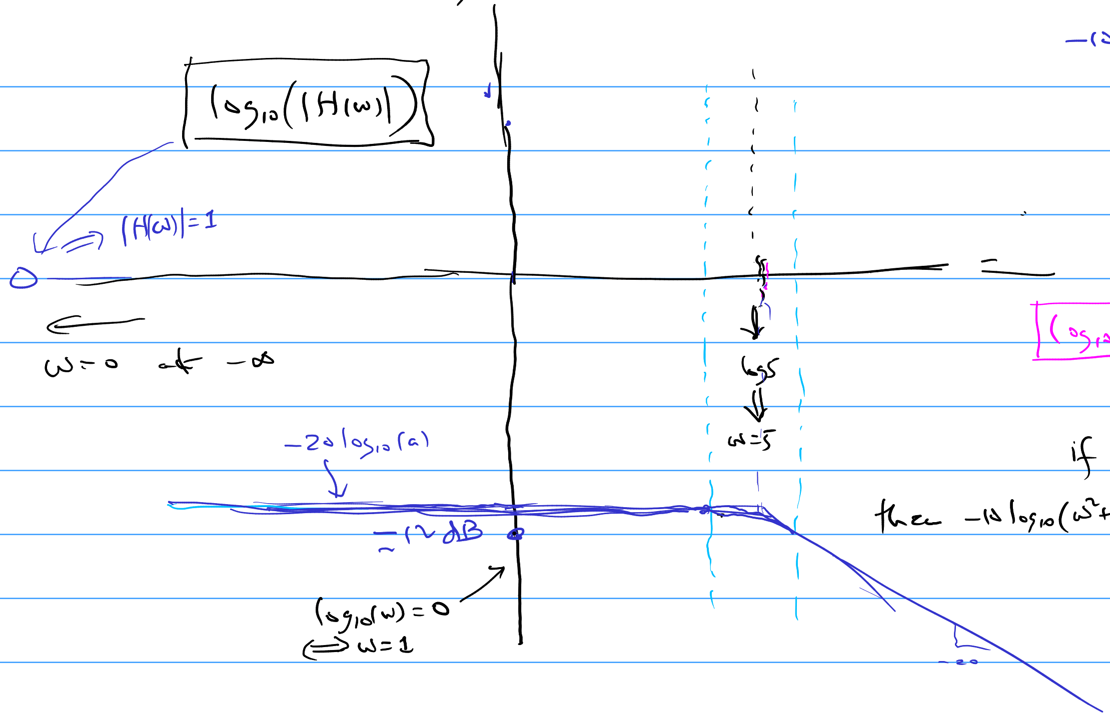
if $(\omega \gg a)$ then

$$-10\log_{10}(\omega^2+a^2) \simeq -10\log_{10}(\omega^2)$$
$$= -20\log_{10}(\omega)$$

$$20\log_{10}(|H(\omega)|) = -10\log_{10}(\omega^2+a^2)$$

$\hookrightarrow 25 = a^2$

$$-10\log_{10}(1^2+a^2)$$



$\omega = a = 5$

$\log(5)$

$\boxed{\log_{10}(\omega)}$

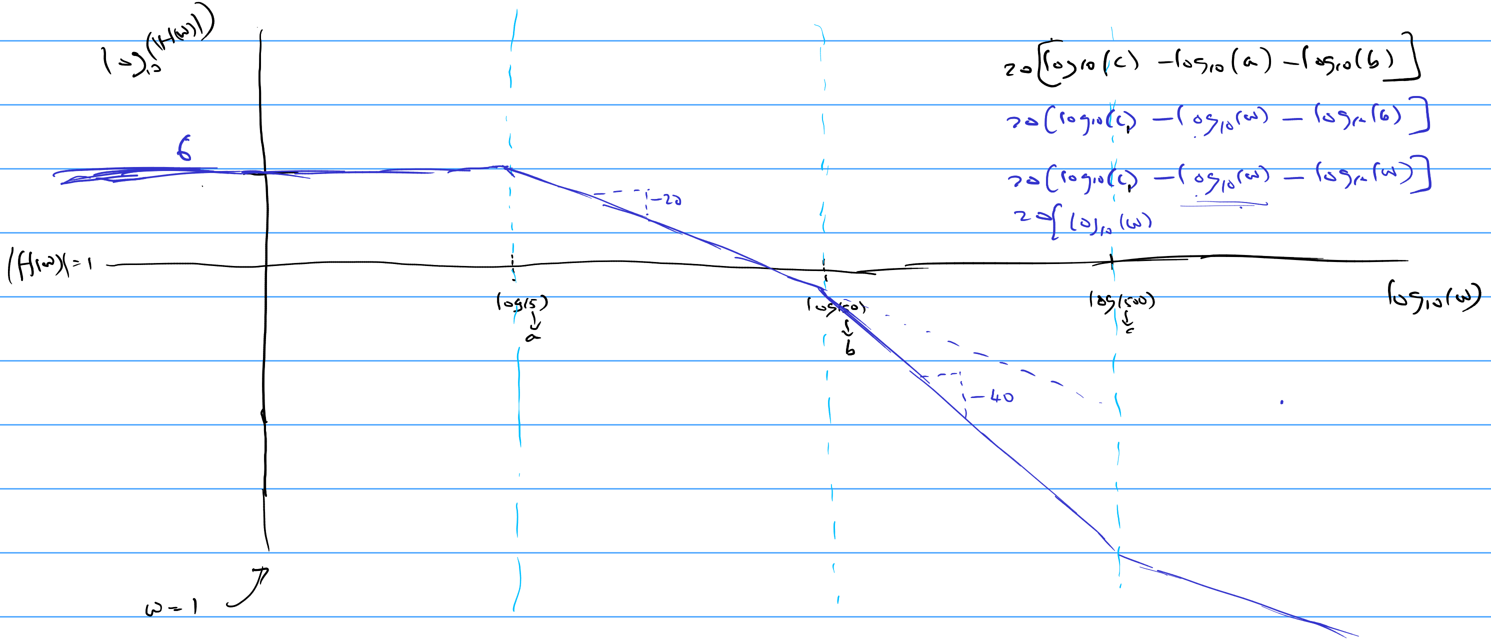
if $\omega \ll a = 5$

then $-10\log_{10}(\omega^2+a^2) \simeq -10\log_{10}(a^2)$

$$= -20\log_{10}(a)$$

$$H(\omega) = \frac{1}{j\omega + \underset{5}{a}} \times \frac{j\omega + \overset{500}{c}}{j\omega + \underset{50}{b}} \Rightarrow |H(\omega)| = \frac{\sqrt{\omega^2 + c^2}}{\sqrt{\omega^2 + a^2}\sqrt{\omega^2 + b^2}}$$

$$20\log_{10}(|H(\omega)|) = 10\left[\log_{10}(\omega^2 + c^2) - \log_{10}(\omega^2 + a^2) - \log_{10}(\omega^2 + b^2)\right]$$



$$20\left[\log_{10}(c) - \log_{10}(a) - \log_{10}(b)\right]$$

$$20\left(\log_{10}(c) - \log_{10}(\omega) - \log_{10}(b)\right]$$

$$20\left(\log_{10}(c) - \log_{10}(\omega) - \log_{10}(\omega)\right]$$

$$20\left[\log_{10}(\omega)\right.$$