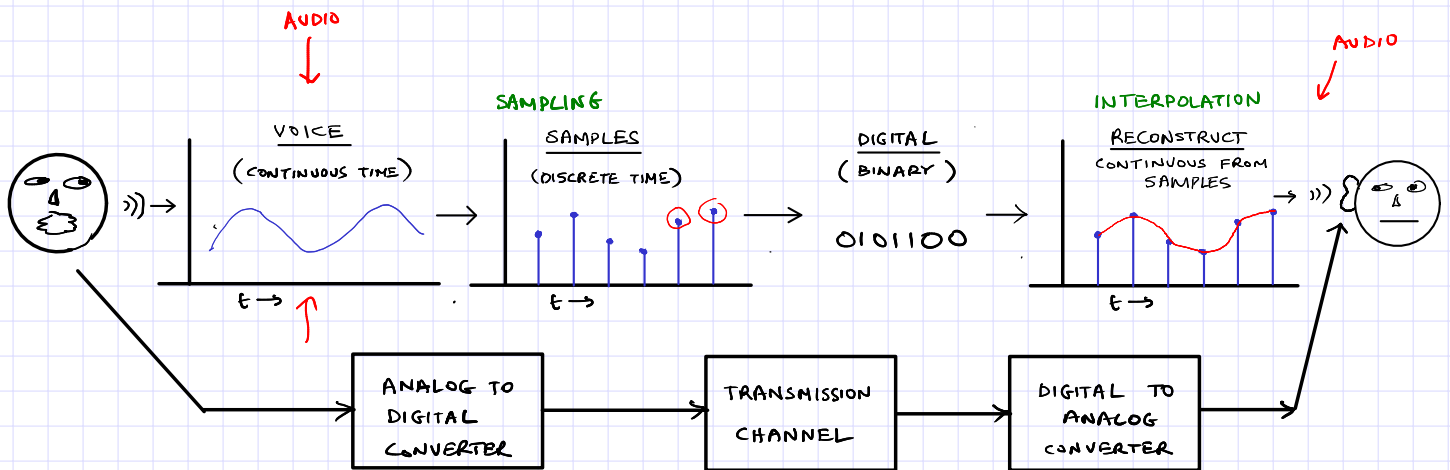


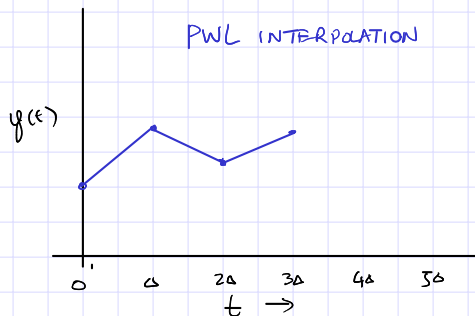
INTERPOLATION

1. MOTIVATION: AUDIO TRANSMISSION EXAMPLE : SAMPLING AND INTERPOLATION
2. PWL AND ZOH INTERPOLATION
3. INTERPOLATION USING BASIS FUNCTIONS
4. SINC INTERPOLATION
5. INTERPOLATION USING (GLOBAL) POLYNOMIALS

1. MOTIVATION

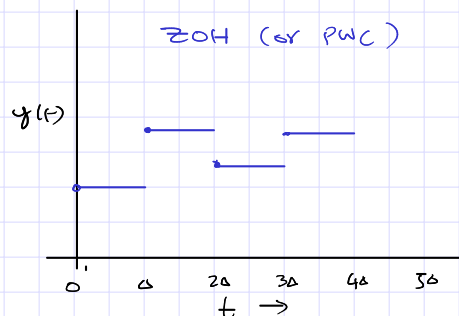


2. PIECEWISE LINEAR (PWL) and ZERO-ORDER HOLD (ZOH) INTERPOLATION



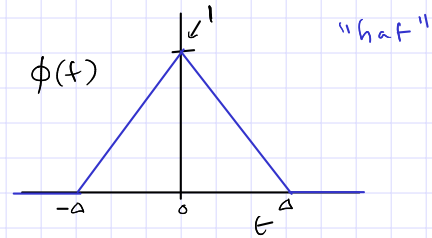
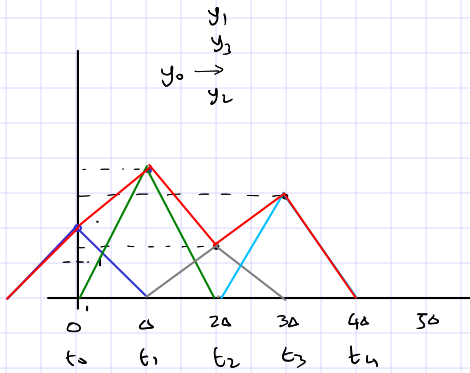
→ CONTINUOUS (NO "JUMPS")

→ DERIVATIVE DISCONTINUOUS
→ HAS "KINKS"



→ "JUMPS" (DISCONTINUITIES)

3. INTERPOLATION USING BASIS FUNCTIONS



$$\phi(t) = \begin{cases} 0 & , t < -\Delta \\ 1 + \frac{t}{\Delta} & , -\Delta \leq t \leq 0 \\ 1 - \frac{t}{\Delta} & , 0 \leq t \leq \Delta \\ 0 & , t > \Delta \end{cases}$$

$$y(t) = y_0 \phi(t-t_0) + y_1 \phi(t-t_1) + y_2 \phi(t-t_2) + y_3 \phi(t-t_3)$$

$$y(t) = \sum_{i=0}^3 y_i \phi(t-t_i) ;$$

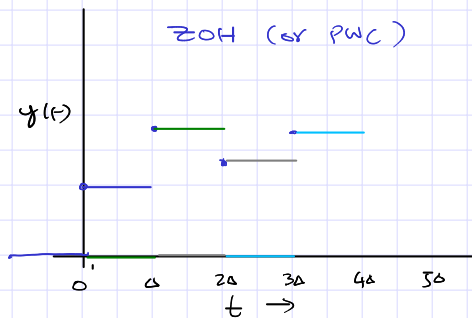
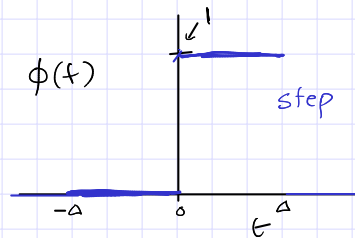
→ Given N samples: $(t_0, y_0) \dots (t_{N-1}, y_{N-1})$

$$y_{PWL}(t) = \sum_{i=0}^{N-1} y_i \phi(t-t_i)$$

$t_0, \dots, t_{N-1}$ are equally spaced at Δ

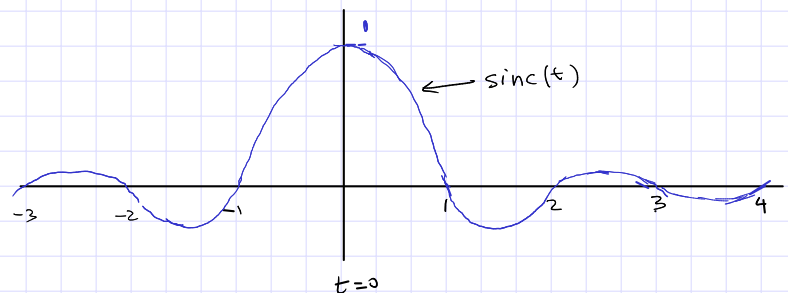
→ THE POWER OF THE BASIS EXPANSION?

$$\rightarrow y_{PWL}(t) = \sum_{i=0}^{N-1} y_i \phi(t-t_i)$$



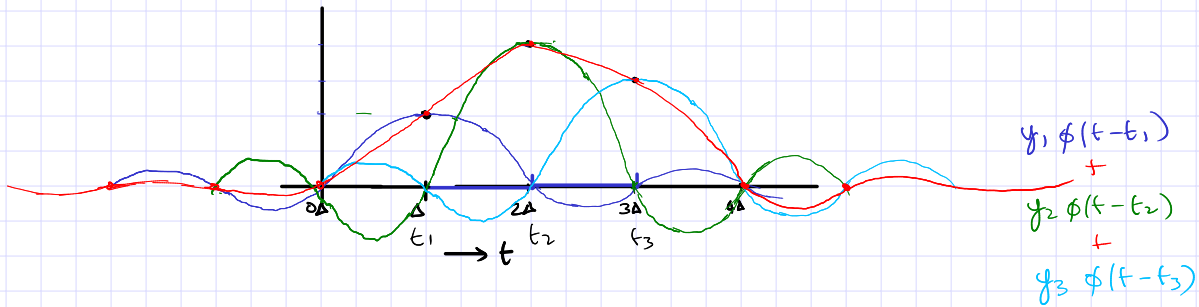
4. SINC INTERPOLATION

$$\rightarrow \text{sinc}(t) \triangleq \begin{cases} 1, & \text{if } t=0 \\ \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \end{cases}$$



$$\phi(t) = \text{sinc}(t/\Delta) \leftarrow$$

$$y(t) = \sum_{i=0}^{N-1} y_i \phi(t-t_i)$$

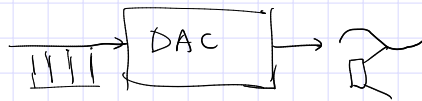


→ ADVANTAGES (OVER PWL & PWC)?

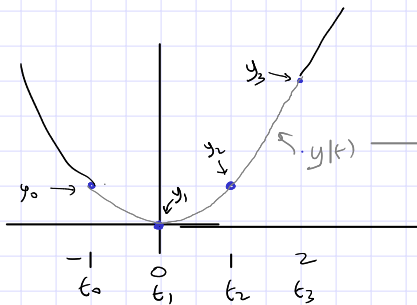
→ "SMOOTH" (NO JUMPS OR LINKS) → ALL DERIVATIVES CONTINUOUS

→ BAND-LIMITED: IMPORTANT

↳ WILL TALK ABOUT IT LATER (WHEN WE DO DFTS)



5. INTERPOLATION USING GLOBAL POLYNOMIALS



DATA: $(-1, 1), (0, 0), (1, 1), (2, 4)$: 4 points = N

$$y(t) = \underline{a}_0 + \underline{a}_1 t + \underline{a}_2 t^2 + \underline{a}_3 t^3$$

$$y_0 = y(t_0) = \underline{a}_0 + \underline{a}_1 t_0 + \underline{a}_2 t_0^2 + \underline{a}_3 t_0^3$$

$$y_1 = y(t_1) = \underline{a}_0 + \underline{a}_1 t_1 + \underline{a}_2 t_1^2 + \underline{a}_3 t_1^3$$

$$y_2 = y(t_2) = \underline{a}_0 + \underline{a}_1 t_2 + \underline{a}_2 t_2^2 + \underline{a}_3 t_2^3$$

$$y_3 = y(t_3) = \underline{a}_0 + \underline{a}_1 t_3 + \underline{a}_2 t_3^2 + \underline{a}_3 t_3^3$$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ 1 & t_3 & t_3^2 & t_3^3 \end{bmatrix} \begin{bmatrix} \underline{a}_0 \\ \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_3 \end{bmatrix}$$

VAN DER MONDE MATRIX
V

IF V is invertible

$$\underline{y} = V \underline{a}$$

$$\underline{a} = V^{-1} \underline{y}$$

- determinant of VAN DER MONDE MATRIX:

$$V = \begin{bmatrix} 1 & t_0 & t_0^2 & \dots & t_0^{N-1} \\ 1 & t_1 & t_1^2 & \dots & t_1^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & t_{N-1} & t_{N-1}^2 & \dots & t_{N-1}^{N-1} \end{bmatrix} \begin{matrix} \uparrow \\ N \end{matrix}$$

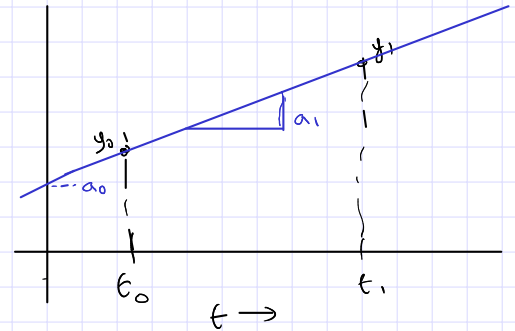
$\leftarrow N$

$$\det(V) = \prod_{0 \leq i < j \leq N-1} (t_j - t_i) \neq 0$$

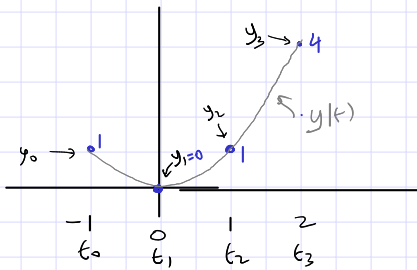
if all timepoints distinct.

→ 2x2 EXAMPLE OF VANDERMONDE MATRIX

$$V = \begin{bmatrix} 1 & t_0 \\ 1 & t_1 \end{bmatrix} \Rightarrow V^{-1} = \frac{1}{t_1 - t_0} \begin{bmatrix} t_1 & -t_0 \\ -1 & 1 \end{bmatrix}$$



$$y(t) = a_0 + a_1 t$$



$$y(t) = t^2 = \begin{matrix} a_0 \\ \downarrow \end{matrix} 0 + \begin{matrix} a_1 \\ \downarrow \end{matrix} 0t + \begin{matrix} a_2 \\ \downarrow \end{matrix} 1t^2 + \begin{matrix} a_3 \\ \downarrow \end{matrix} 0t^3$$

→ ADVANTAGES OF POLYNOMIAL INTERPOLATION (OVER BASIS FN INTERP.)?