→ LTI SYSTEMS AND DFT: OVERVIEW*

LTI SYSTEMS

IMPULSE RESPONSE $h[t]$ CHARACTERISES $u[t] \mapsto y[t]$ FULLY VIA CONVOLUTION:

$$y[t] = u[t] \circledast h[t]$$

IF YOU ZERO-PAD $\vec{u}$ and $\vec{h}$ (TO $\vec{u}_p$ and $\vec{h}_p$), then CIRCULAR CONVOLUTION = CONVOLUTION

i.e., $\vec{u} \circledast \vec{h} = \vec{u}_p \circledast_c \vec{h}_p$

DFT

- COMPLEX CONJUGACY: $\vec{x}$ REAL $\Leftrightarrow X_{N-i} = \overline{X}_i$
- DFT TRANSFORMS T.D. TO F.D.: i.e., SINUSOID AT FREQ. $\frac{k}{N} \Rightarrow X_k$ = (SCALED) PHASOR
- DFT COEFFS. X_k CAN BE FILTERED EASILY: $Y_k = X_k H_k$: Y_k ARE FILTERED DFT COEFFS.

CIRCULAR CONVOLUTION IN T.D. = ELEM. BY ELEM. MULTIPLICATION IN F.D.

→ i.e., $\vec{x} \circledast_c \vec{y} = F_N^{-1}\left((F_N \vec{x}) .* (F_N \vec{y})\right)$

⇒ CIRCULAR CONVOLUTION CAN BE DONE FAST (VIA FFT)

* SIMPLIFIED: SEE LECTURES/NOTES FOR DETAILS

DFT CAN BE USED FOR FAST CONVOLUTION VIA ZERO-PADDING

→ EXAMPLE: CONVOLUTION VIA PADDING AND DFT

$M=2$, $N=2M+1=5$

$\vec{u} = [1\ 0\ 1\ 0\ 1]^T$

$\vec{h} = [5\ 4\ 3\ 2\ 1]^T$

$\vec{y} = \vec{u} \circledast \vec{h} = \vec{y} = [5\ 4\ 8\ 6\ 9]^T$

		1	0	1	0	1	← u[i]
		1	2	3	4	5	← h[t-i]
1	2	3	4	5			← h[-i]

$\vec{u}_p = [1\ 0\ 1\ 0\ 1\ \overbrace{0\ 0\ 0\ 0}^{N-1 \text{ ZEROS}}] \Rightarrow \vec{U}_p = F_9 \vec{u}_p =$ DO IT IN MATLAB

$\vec{h}_p = [5\ 4\ 3\ 2\ 1\ 0\ 0\ 0\ 0] \Rightarrow \vec{H}_p = F_9 \vec{h}_p =$ DO IT IN MATLAB

$\vec{Y}_p \triangleq \vec{U}_p .* \vec{H}_p =$ DO IT IN MATLAB

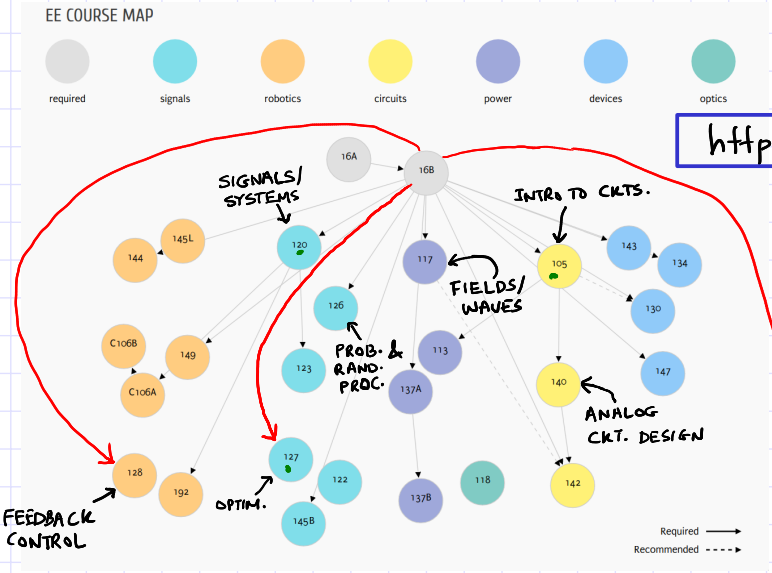
$\vec{y}_p = F_9^{-1} \vec{Y}_p =$ DO IN MATLAB

→ THE FIRST N NUMBERS MATCH $\vec{y}$.

→ WHAT IF YOU DON'T ZERO PAD?

→ $\vec{U} = F_5 \vec{u}$; $\vec{H} = F_5 \vec{h}$, $\vec{Y} = \vec{U} .* \vec{H}$; $\vec{y} \overset{?}{=} F_5^{-1} \vec{Y}$? DO IN MATLAB

→ NO: COMPLETELY DIFFERENT: $F_5^{-1} \vec{Y} = [11\ 8\ 10\ 7\ 9]^T$



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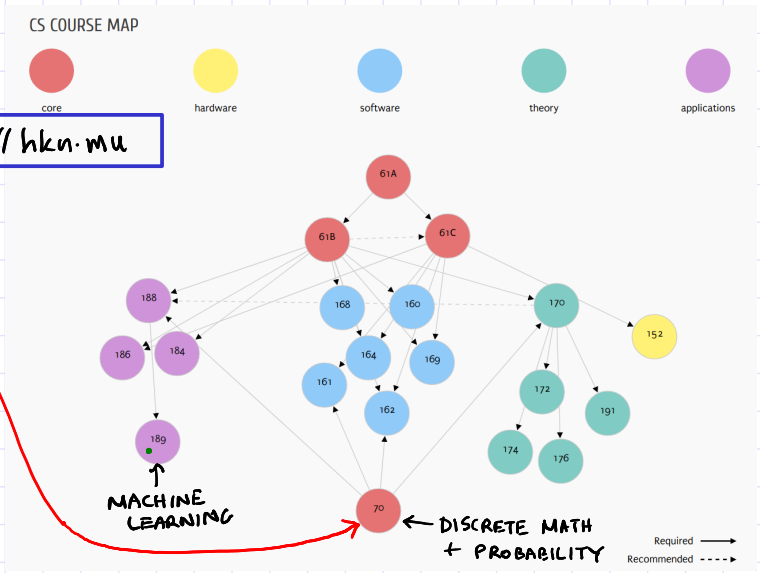

→ 128 (FEEDBACK CONTROL): STABILIZING INVERTED PENDULUM ON A CART (LAB); STATE SPACE, CONTROLS

→ 120 (SIGNALS & SYSTEMS): (16B SIGNALS/SYSTEMS)++: C.T. SIGNALS/SYSTEMS; LIN. ALG., SIGNALS, SYSTEMS, DFT

→ 127 (OPTIMIZATION): LINEAR & NONLINEAR OPTIMIZATION; SVDs, PCA, LEAST SQUARES. SERIOUS LIN. ALG.

→ 126 (PROBABILITY AND RANDOM PROCESSES): NOISE, MARKOV CHAINS; SIGNALS/SYSTEMS, STATE SPACE, DFT

→ 117 (ELECTROMAGNETIC FIELDS & WAVES): MAXWELL'S EQUATIONS, ANTENNAS; STATE-SPACE, EIGEN, CIRCUITS

→ 105 and 140 (CIRCUITS): HOW TO ANALYZE AND DESIGN REAL CIRCUITS W TRANSISTORS; CIRCUITS, FEEDBACK/CONTROLS

→ 70 (DISCRETE MATH & PROB): BASIC MATH UNDERLYING CS; MATH REASONING/PROOFS, EIGENDECOMPOSITION