→ LTI SYSTEMS AND DFT: OVERVIEW*

LTI SYSTEMS

IMPULSE RESPONSE $h[t]$ CHARACTERISES $u[t] \mapsto y[t]$ FULLY VIA CONVOLUTION:

$$y[t] = u[t] \circledast h[t]$$

IF YOU ZERO-PAD $\vec{u}$ and $\vec{h}$ (TO $\vec{u}_p$ and $\vec{h}_p$), then CIRCULAR CONVOLUTION = CONVOLUTION

i.e., $\vec{u} \circledast \vec{h} = \vec{u}_p \circledast_c \vec{h}_p$

DFT

- COMPLEX CONJUGACY: $\vec{x}$ REAL $\Leftrightarrow X_{N-i} = \overline{X}_i$
- DFT TRANSFORMS T.D. TO F.D.: i.e., SINUSOID AT FREQ. $\frac{k}{N} \Rightarrow X_k$ = (SCALED) PHASOR
- DFT COEFFS. X_k CAN BE FILTERED EASILY: $Y_k = X_k H_k$: Y_k ARE FILTERED DFT COEFFS.

CIRCULAR CONVOLUTION IN T.D. = ELEM. BY ELEM. MULTIPLICATION IN F.D.

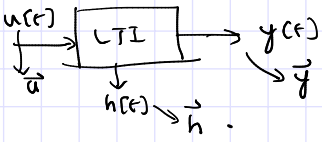
→ i.e., $\vec{x} \circledast_c \vec{y} = F_N^{-1}\left((F_N \vec{x}) \cdot * (F_N \vec{y})\right)$

⇒ CIRCULAR CONVOLUTION CAN BE DONE FAST (VIA FFT)

* SIMPLIFIED: SEE LECTURES/NOTES FOR DETAILS

DFT CAN BE USED FOR FAST CONVOLUTION VIA ZERO-PADDING

→ EXAMPLE: CONVOLUTION VIA PADDING AND DFT



→ $M = 2$, $N = 2M+1 = 5$

→ $\vec{u} = [1\ 0\ 1\ 0\ 1]^T$

→ $\vec{h} = [5\ 4\ 3\ 2\ 1]^T$

→ $\vec{y} = \vec{u} \circledast \vec{h} = [5\ 4\ 8\ 6\ 9]^T$

$$\sum_{i=0}^{t} u[i]\,h[t-i]$$

				1	0	1	0	1	← $u[i]$
				1	2	3	4	5	$h[t-i]$
1	2	3	4	5					← $h[-i]$ ⇒ $t=0$
-4	-3	-2	-1	0	1	2	3	4	← $[i]$
				5	4	8	6	9	← $y[i]$

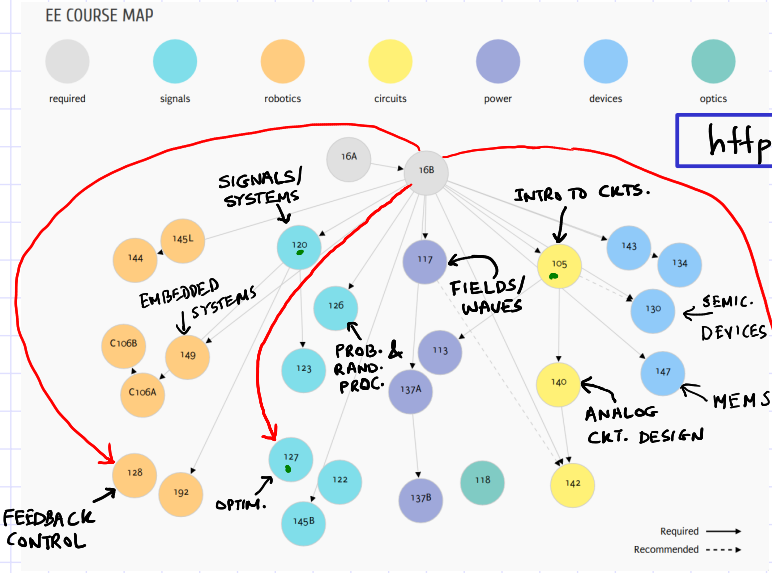
→ $\vec{u}_p = [1\ 0\ 1\ 0\ 1\ \overbrace{0\ 0\ 0\ 0}^{N-1}]^T$ → $\vec{U}_p = F_9 \vec{u}_p$

→ $\vec{h}_p = [5\ 4\ 3\ 2\ 1\ 0\ 0\ 0\ 0]^T$ → $\vec{H}_p = F_9 \vec{h}_p$

$\vec{Y}_p = \vec{U}_p \cdot * \vec{H}_p$

$\vec{y}_p = F_9^{-1} \vec{Y}_p$

→ $\vec{U} = F_5 \vec{u}$, $\vec{H} = F_5 \vec{h}$, $\vec{Y} = \vec{U} \cdot * \vec{H}$, $F_5^{-1} Y = ?$ $\vec{y}$?



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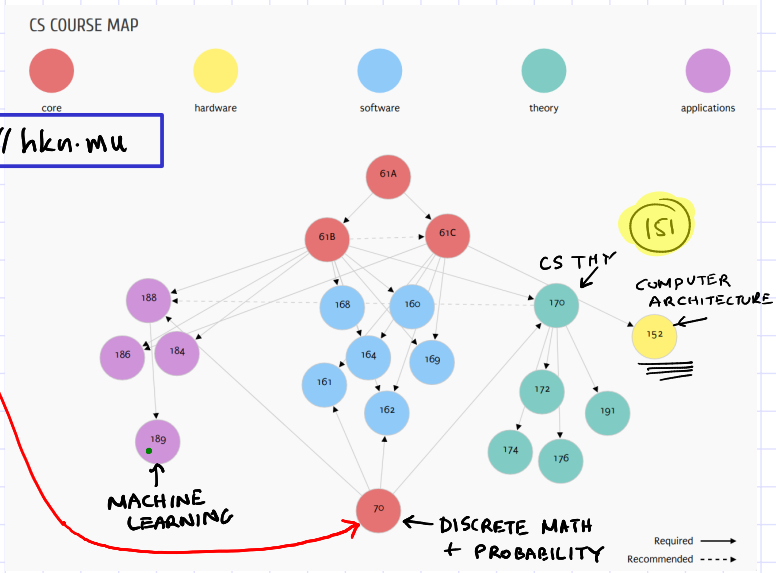


→ 128 (FEEDBACK CONTROL): STABILIZING INVERTED PENDULUM ON A CART (LAB); STATE SPACE, CONTROLS

→ 120 (SIGNALS & SYSTEMS): (16B SIGNALS/SYSTEMS)++: C.T. SIGNALS/SYSTEMS; LIN. ALG., SIGNALS, SYSTEMS, DFT

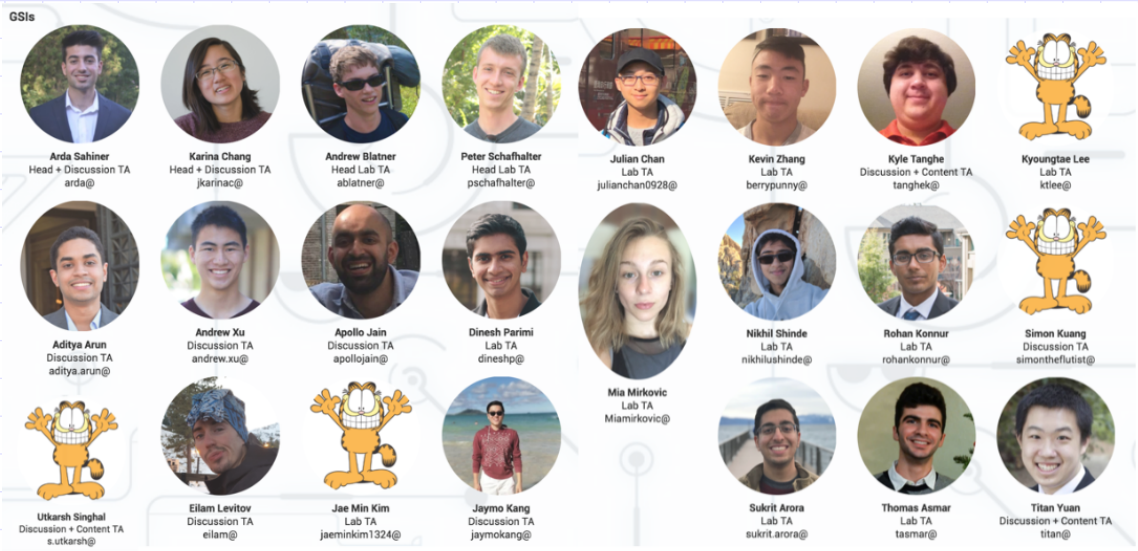
→ 127 (OPTIMIZATION): LINEAR & NONLINEAR OPTIMIZATION; SVDs, PCA, LEAST SQUARES. SERIOUS LIN. ALG.

→ 126 (PROBABILITY AND RANDOM PROCESSES): NOISE, MARKOV CHAINS; SIGNALS/SYSTEMS, STATE SPACE, DFT

→ 117 (ELECTROMAGNETIC FIELDS & WAVES): MAXWELL'S EQUATIONS, ANTENNAS; STATE-SPACE, EIGEN, CIRCUITS

→ C05 and 140 (CIRCUITS): HOW TO ANALYZE AND DESIGN REAL CIRCUITS W TRANSISTORS; CIRCUITS, FEEDBACK/CONTROLS

→ 70 (DISCRETE MATH & PROB): BASIC MATH UNDERLYING CS; MATH REASONING/PROOFS, EIGENDECOMPOSITION



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