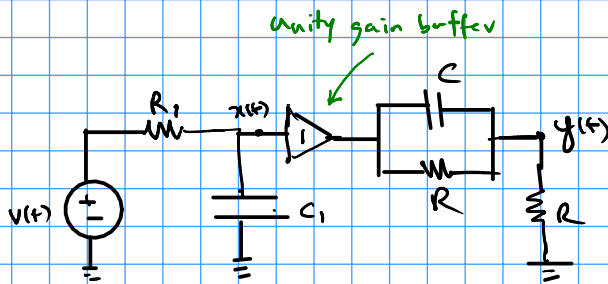


BODE PLOTS FOR SYSTEMS WITH MORE THAN 1 POLE/ZERO

→ EXAMPLE CIRCUIT WITH 2 POLES AND 1 ZERO (ALL REAL)



$$\tilde{X} = \frac{\tilde{V}}{1 + j\omega R_1 C_1} ; \tilde{Y} = \frac{\tilde{X} R}{R + (R \parallel Z_C)} = \frac{\tilde{X} R}{R + \frac{R Z_C}{R + Z_C}} = \frac{\tilde{X}}{1 + \frac{Z_C}{R + Z_C}} = \frac{\tilde{X} (R + Z_C)}{R + 2Z_C}$$

$$= \frac{\tilde{X} (R + \frac{1}{j\omega C})}{R + \frac{2}{j\omega C}} = \frac{\tilde{X} (1 + j\omega RC)}{2 + j\omega RC}$$

$$\Rightarrow \tilde{Y} = \tilde{V} \frac{1}{2} \frac{(1 + j\omega RC)}{(1 + j\omega R_1 C_1)(1 + j\omega \frac{RC}{2})}$$

$$= \tilde{V} \frac{1}{2} \frac{RC(j\omega + \frac{1}{RC})}{R_1 C_1 (j\omega + \frac{1}{R_1 C_1}) \frac{RC}{2} (j\omega + \frac{1}{(RC/2)})}$$

call this z_1

call this p_1

call this p_2

$$= \tilde{V} \frac{RC}{(R_1 C_1)(RC)} \frac{(j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)}$$

call this K

→ TRANSFER FN. $H(\omega) = \frac{\tilde{Y}}{\tilde{V}}$ HAS THE FORM:

$$\frac{K (j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)}$$

zero #1

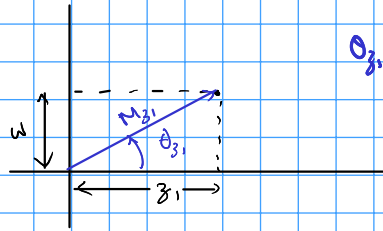
pole #1

pole #2

→ BODE PLOT OF $H(\omega) \triangleq \frac{K (j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)}$

→ First, express each pole/zero term in polar co-ordinates

→ $j\omega + z_1$:



$$\theta_{z_1} = \tan^{-1}\left(\frac{\omega}{z_1}\right) ; M_{z_1} = \sqrt{\omega^2 + z_1^2}$$

$$\Rightarrow j\omega + z_1 = M_{z_1} e^{j\theta_{z_1}}$$

→ Similarly: $j\omega + p_1 = M_{p_1} e^{j\theta_{p_1}}$, $M_{p_1} = \sqrt{\omega^2 + p_1^2}$, $\theta_{p_1} = \tan^{-1}\left(\frac{\omega}{p_1}\right)$

$$j\omega + p_2 = M_{p_2} e^{j\theta_{p_2}} , M_{p_2} = \sqrt{\omega^2 + p_2^2} , \theta_{p_2} = \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

$$\Rightarrow H(\omega) = \frac{K M_{z_1}(\omega) e^{j\theta_{z_1}(\omega)}}{M_{p_1}(\omega) e^{j\theta_{p_1}(\omega)} M_{p_2}(\omega) e^{j\theta_{p_2}(\omega)}}$$

$$\Rightarrow H(\omega) = \frac{K M_{z_1}(\omega)}{M_{p_1}(\omega) M_{p_2}(\omega)} e^{j[\theta_{z_1}(\omega) - \theta_{p_1}(\omega) - \theta_{p_2}(\omega)]}$$

$$\Rightarrow |H(\omega)| = \frac{K M_{z_1}(\omega)}{M_{p_1}(\omega) M_{p_2}(\omega)}$$

$$\angle H(\omega) = \theta_{z_1}(\omega) - \theta_{p_1}(\omega) - \theta_{p_2}(\omega)$$

$$\log_{10}(|H(\omega)|) = \log_{10}(K) + \log_{10}(M_{z_1}(\omega)) - \log_{10}(M_{p_1}(\omega)) - \log_{10}(M_{p_2}(\omega))$$

↑
+ for zero
↑
- for pole
↑
- for pole

→ LOOK AT THE TERMS :

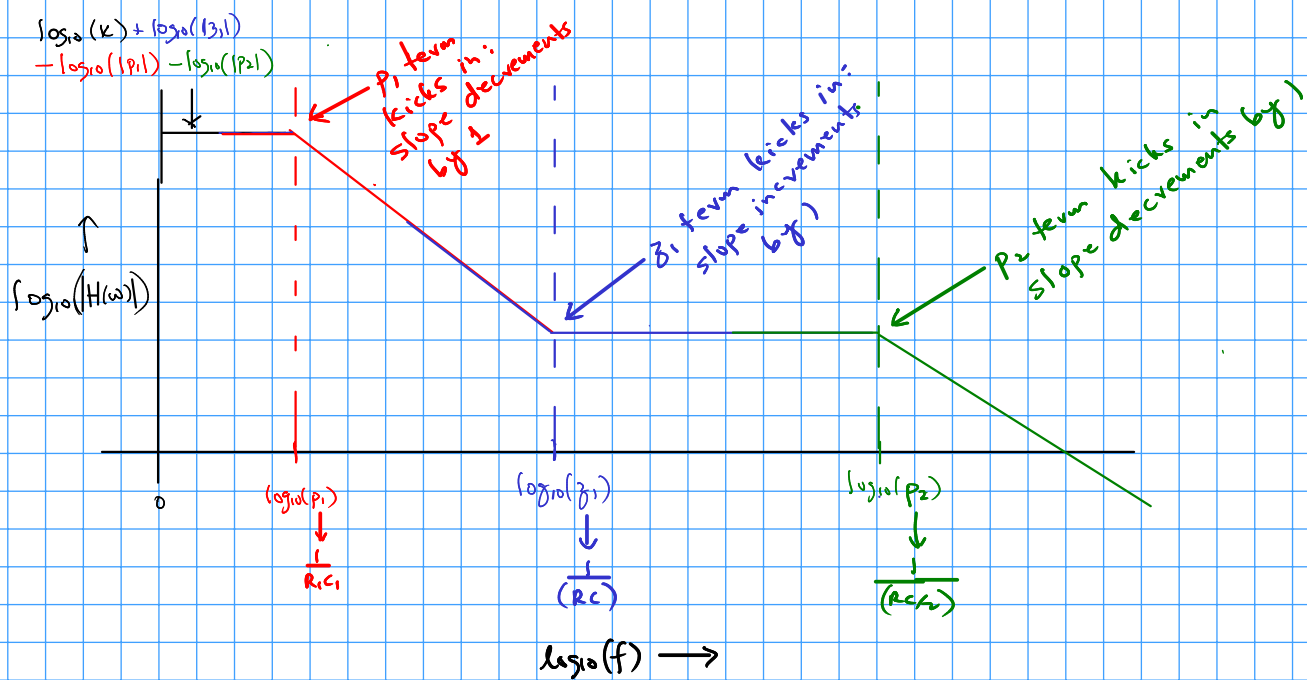
→ $\log_{10}(K)$: constant for all ω

→ $+\log_{10}(M_{z_1}(\omega))$: $+\log_{10}(|z_1|)$ for $\omega \ll z_1$, $+\log_{10}(\omega)$ for $\omega \gg z_1$

→ $-\log_{10}(M_{p_1}(\omega))$: $-\log_{10}(|p_1|)$ for $\omega \ll p_1$, $-\log_{10}(\omega)$ for $\omega \gg p_1$

→ $-\log_{10}(M_{p_2}(\omega))$: $-\log_{10}(|p_2|)$ for $\omega \ll p_2$, $-\log_{10}(\omega)$ for $\omega \gg p_2$

→ AND THE TERMS ADD, MAKING PLOTTING EASY



→ PLOTTING THE ANGLE $\angle H(\omega) = \Theta_{z_1}(\omega) - \Theta_{p_1}(\omega) - \Theta_{p_2}(\omega)$

→ LOOK AT EACH TERM:

→ $\Theta_{z_1}(\omega)$: 0° for $\omega \ll z_1$, $+90^\circ$ for $\omega \gg z_1$, $+45^\circ$ at $\omega = z_1$.

→ $-\Theta_{p_1}(\omega)$: 0° for $\omega \ll p_1$, -90° for $\omega \gg p_1$, -45°

→ $-\Theta_{p_2}(\omega)$: 0° for $\omega \ll p_2$, -90° for $\omega \gg p_2$, -45°

