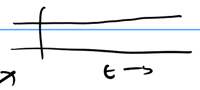
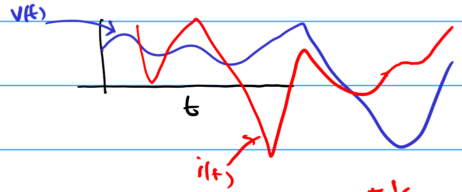
→ RECAP

→ 3 DIFFERENT TYPES OF CIRCUIT ANALYSIS



→ DC: all voltages / currents unchanging w time ← COVERED IN 16A



→ SIMPLE ALGEBRAIC EQNS: $i = \frac{V}{R}$, $v_2 = \frac{R_2}{R_1+R_2} v_1$

→ transient : " " CHANGING WITH TIME

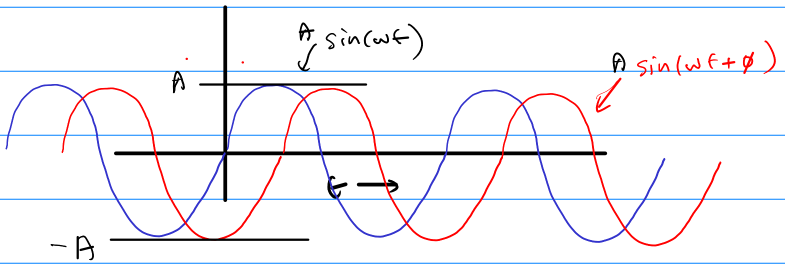
→ DIFFERENTIAL EQNS. : $i(t) = C\frac{dv}{dt}$, $C\frac{dv(t)}{dt} = -\frac{v(t)}{R}$ $\left(\sim \frac{dv}{dt} = \overset{-1/RC}{\lambda}\, v(t)\right)$

→ SOLUTION $v(t) = V_0 e^{\lambda t}$ ← INITIAL CONDITION : $V_0 = v(0)$

↳ ALL YOU NEED TO KNOW ODES

KRIS: SHOWED THIS SOLVES ODE ← HW: SHOWS ONLY THIS SOLVES IT

→ SPECIAL CASE : everything changing SINUSOIDALLY with time



→ SPECIAL TYPE OF CONVENIENT ANALYSIS: SINUSOIDAL S. STATE

→ PHASORS

→ FREQ. DOMAIN TRANSFER FUNCTIONS

→ ALL SOLVING THE SAME CKT / PROBLEM

→ IN DIFFERENT WAYS

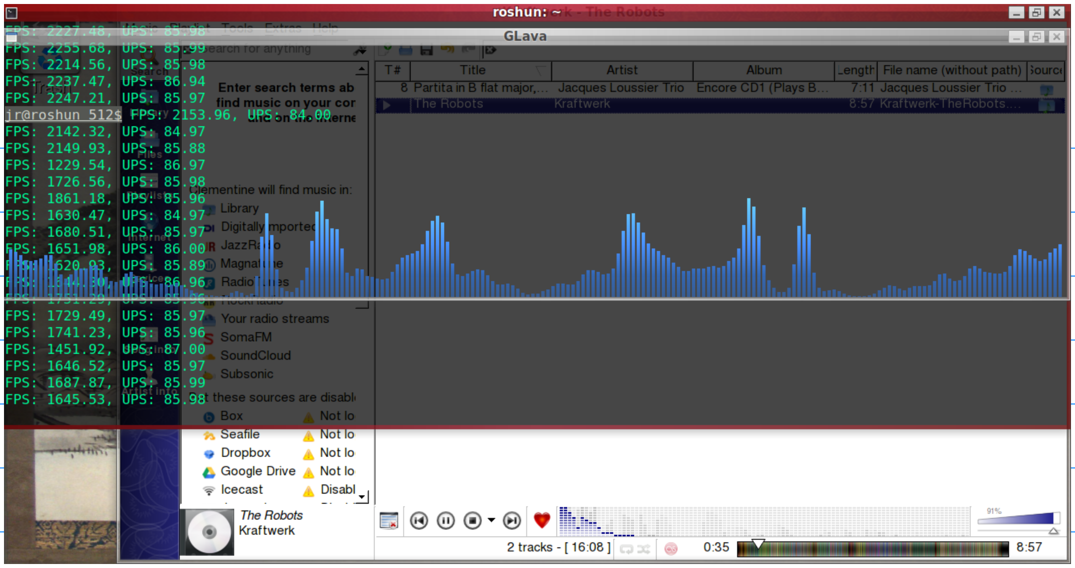
→ TRANSIENT: MOST GENERAL (BUT MOST INVOLVED)

→ DC : EASY AND IMPORTANT (BUT LIMITED USES)

→ SINUSOIDAL STEADY-STATE : ALMOST - TRANSIENT (BUT EASIER AND MORE INSIGHTFUL THAN TRANSIENT)

→ WHY ARE SINUSOIDS IMPORTANT / INTERESTING

→ DEMO



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FPS: 2255.68, UPS: 85.99
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FPS: 2237.47, UPS: 86.94
FPS: 2247.21, UPS: 85.97
jr@roshun 512$ FPS: 2153.96, UPS: 84.00
FPS: 2142.32, UPS: 84.97
FPS: 2149.93, UPS: 85.88
FPS: 1229.54, UPS: 86.97
FPS: 1726.56, UPS: 85.98
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FPS: 1680.51, UPS: 85.97
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FPS: 1644.30, UPS: 86.96
FPS: 1751.29, UPS: 85.96
FPS: 1729.49, UPS: 85.97
FPS: 1741.23, UPS: 85.96
FPS: 1451.92, UPS: 87.00
FPS: 1646.52, UPS: 85.97
FPS: 1687.87, UPS: 85.99
FPS: 1645.53, UPS: 85.98
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Enter search terms above to find music on your computer and on the internet

Clementine will find music in:
- Library
- Digitally Imported
- JazzRadio
- Magnatune
- RadioTunes
- RockRadio
- Your radio streams
- SomaFM
- SoundCloud
- Subsonic

But these sources are disabled:

Box	Not lo
Seafile	Not lo
Dropbox	Not lo
Google Drive	Not lo
Icecast	Disabl

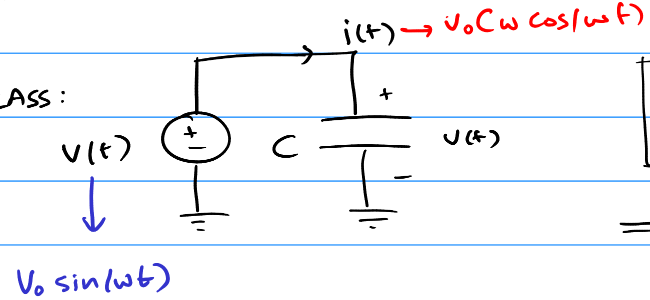
→ VOICE / MUSIC / etc CAN BE REPRESENTED AS A SUM OF MANY SINUSOIDS (AT DIFFERENT FREQS.)

→ CHANGING THE RELATIVE STRENGTHS OF DIFFERENT SINUSOIDS: FILTERING

→ SINUSOIDAL ANALYSIS: STEP TOWARDS UNDERSTANDING THIS.

→ LATER: DFT ; LATER: EE 120

→ LAST CLASS:



$$\boxed{i(t) = C\frac{dv}{dt}}$$

$$\Rightarrow i(t) = C\frac{d}{dt}\left(v_0 \sin(\omega t)\right)$$

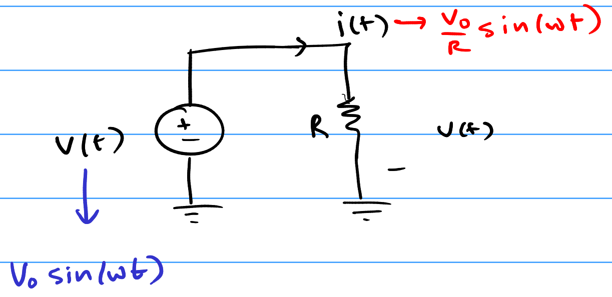
$$\Rightarrow \boxed{i(t) = V_0 C\omega \cos(\omega t)}$$

→ COMPARE w RESISTOR

$$\boxed{i(t) = \frac{v(t)}{R}}$$

$$\downarrow$$

$$\Rightarrow \boxed{R = \frac{v(t)}{i(t)}}$$



CHECK: $\dfrac{V_0 \sin(\omega t)}{\frac{V_0}{R}\sin(\omega t)} = R$ ✓

→ WHAT IS $\frac{v(t)}{i(t)}$ FOR A CAP. WITH SIN. INPUT?

CAP : $\frac{v(t)}{i(t)} = \frac{\cancel{V_0}\sin(\omega t)}{\cancel{V_0}C\omega\cos(\omega t)} = \underset{\uparrow \text{ depends on input freq?}}{\frac{1}{C\omega}}\tan(\omega t)$ ← ???

↳ "resistance" depends on time?

↳ goes to ∞ ?

↳ becomes −ve ?

→ SEEMS TO COMPLICATED COMPARED TO RES.

→ CAN WE DO BETTER?

* * *

→ THE KEY (as it turns out): express sin()/cos() USING

COMPLEX NUMBERS

→ RESOURCES: → TUTORIAL by TAs (up last night)

→ short recap: this week's discussion.

→ very short RECAP:

→ $j = \sqrt{-1}$: imaginary number ⟹ $j^2 = -1$

→ $a = \underset{\substack{\uparrow\\ \text{real part}}}{a_r} + j\underset{\substack{\uparrow\\ \text{imag. part}}}{a_i}$: complex number

→ follows usual rules of addition/mult./etc:

→ $c = a \pm b = (a_r + j a_i) \pm (b_r + j b_i)$

$= \underbrace{(a_r \pm b_r)}_{c_r} + j\underbrace{(a_i \pm b_i)}_{c_i}$

→ CONJUGATE of complex numbers:

Given $a = a_r + j a_i$

conjugate → $\bar{a} \triangleq a_r - j a_i$ (change sign of imag. part)

(unrelated to logical NOT)

→ $a + \bar{a}$ = ALWAYS REAL! ← V. IMP.

→ $(a_r + ja_i) + (a_r - ja_i) = \underset{\uparrow \text{real}}{\underline{2a_r}}$

→ $a - \bar{a}$ = ALWAYS IMAG. = $2ja_i$

EXTREMELY IMP.

→ RELATION TO sin()/cos(): de MOIVRE'S THM. ↖"dMT"

$$\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$$

→ USING dMT, can write sin()/cos() using $e^{j\theta}$ & $e^{-j\theta}$:

$$\to \quad e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$+ \quad e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

[using $\cos(-\theta) = \cos(\theta)$ & $\sin(-\theta) = -\sin(\theta)$]

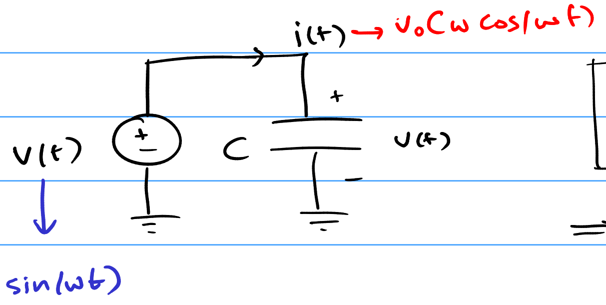
$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\Longrightarrow \boxed{\cos(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2}}$$

→ SUBTRACT: $\boxed{\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$

WILL USE THESE

→



$$\boxed{i(t) = C\frac{dv}{dt}}$$

$$\Longrightarrow i(t) = C\frac{d}{dt}(v_0 \sin(\omega t))$$

$$\to \quad V_0 \sin(\underbrace{\omega t}_{\theta(t)}) = \frac{V_0}{2j}\left[e^{j\omega t} - e^{-j\omega t}\right] \Longrightarrow i(t) = C\frac{d}{dt}\Big[\left[e^{j\omega t} - e^{-j\omega t}\right]\Big]$$

$$\Longrightarrow i(t) = \frac{V_0}{2\cancel{j}} C\cancel{j}\omega\left[e^{j\omega t} + e^{-j\omega t}\right] = V_0 \omega C \times \underbrace{\frac{1}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]}_{\to \cos(\omega t)}$$

→ FOR THE CAP w SIN. INPUT:

$$v(t) = \frac{V_0}{2j} e^{j\omega t} - \frac{V_0}{2j} e^{-j\omega t}$$

$\frac{V_0}{2j} \to \tilde{V}$

$$i(t) = \frac{V_0 C\omega}{2} e^{j\omega t} + \frac{V_0 C\omega}{2} e^{-j\omega t}$$

$\frac{V_0 C\omega}{2} \to \tilde{I}$

→ LOOK AT JUST THE COEFFS OF $e^{j\omega t}$

PHASOR OF i(t)

$$\tilde{V} = \frac{V_0}{2j}, \quad \tilde{I} = \frac{V_0 C\omega}{2}$$

PHASOR OF v(t)

PHASORS

COMPLEX CONJUGATES

$$v(t) = \tilde{V} e^{j\omega t} + \bar{\tilde{V}} e^{-j\omega t} \qquad i(t) = \tilde{I} e^{j\omega t} + \bar{\tilde{I}} e^{-j\omega t}$$

COMPLEX CONJUGATES

→ COMPLEX CONJUGATES!

↳ SO: THEY ALWAYS ADD TO REAL

→ v(t)/i(t) ALWAYS REAL AND **NEVER COMPLEX!**

↑ NEVER!

→ BUT THEIR PHASORS ARE COMPLEX

→ WHY ARE PHASORS USEFUL/INTERESTING?

→ TRY $$\frac{\tilde{V}}{\tilde{I}} = \frac{\frac{\cancel{V_0}}{2j}}{\frac{\cancel{V_0} C\omega}{\cancel{2}}} = \frac{1}{j\omega C}$$

→ Like a resistance, but depends on ω

↓ angular freq.

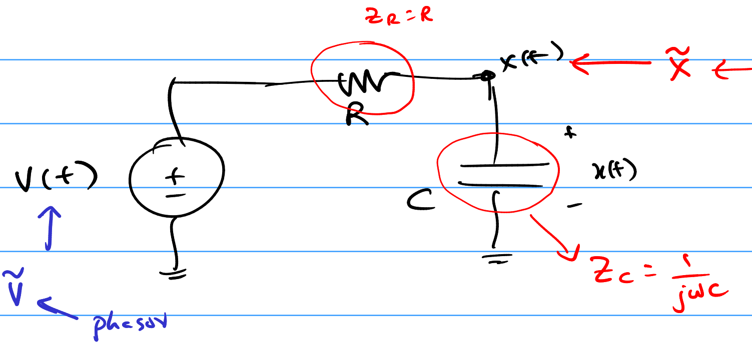
↑ "IMPEDANCE" OF CAPACITOR

↙ Z_C

→ RELATES v & i PHASORS

$$\boxed{\tilde{I} = \frac{\tilde{V}}{Z_C}}$$

→ YOU CAN ANALYSE CIRCUITS WITH PHASORS!



← LIKE A RESISTIVE VOLTAGE DIVIDER
↑
16A

$$\tilde{X} = \tilde{V}\frac{Z_C}{R+Z_C} = \tilde{V}\left(\frac{\frac{1}{j\omega C}}{R+1/j\omega C}\right) = \tilde{V}\frac{1}{1+j\omega RC}$$

→ $\tilde{X} = \tilde{V} \times \frac{1}{1+j\omega RC}$

voltage phasors.

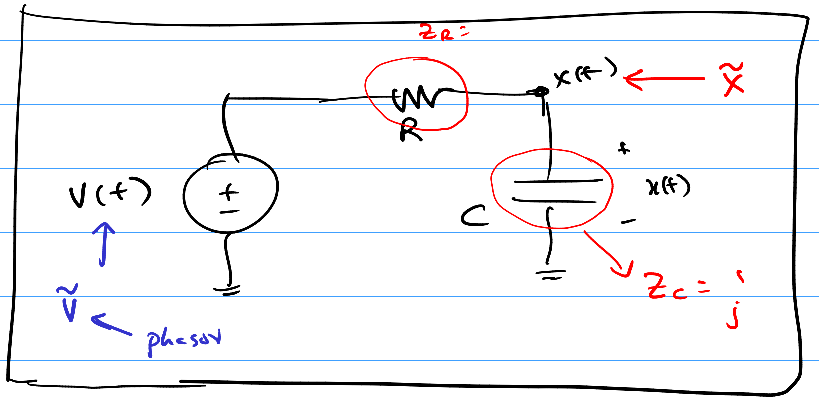
→ $\boxed{\frac{\tilde{X}}{\tilde{V}}} = \frac{1}{1+j\omega RC}$

H(ω)

VOLTAGE TRANSFER FUNCTION

FREQ. DOMAIN (≡ PHASOR DOMAIN)

→ ENABLES A SYSTEM VIEW OF THE CKT.



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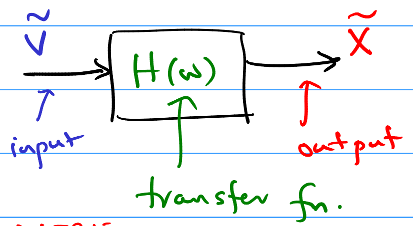


OUTPUT = (XFER-FN) × INPUT

$$\boxed{\tilde{X} = \underbrace{H(\omega)}_{\frac{1}{1+j\omega RC}}\tilde{V}}$$

PHASORS MAKE THIS POSSIBLE!

→ WHAT HAVE WE ACHIEVED?

→ GIVEN SINUSOIDAL $v(t) = \underbrace{\tilde{V}}_{\text{phasor}} e^{j\omega t}$ + c.c. term

phasor ← given this

→ FOUND $\tilde{X}$ = phasor for x(t) = $\tilde{V}\,H(\omega) = \dfrac{\tilde{V}}{1+j\omega RC}$

→ Now, we can find x(t) very easily.

→ $x(t) = \tilde{X}e^{j\omega t}$ + c.c. term.

$= \boxed{\dfrac{\tilde{V}}{1+j\omega RC}} e^{j\omega t}$ + c.c. term → $\dfrac{\bar{\tilde{V}}}{1-j\omega RC} e^{-j\omega t}$

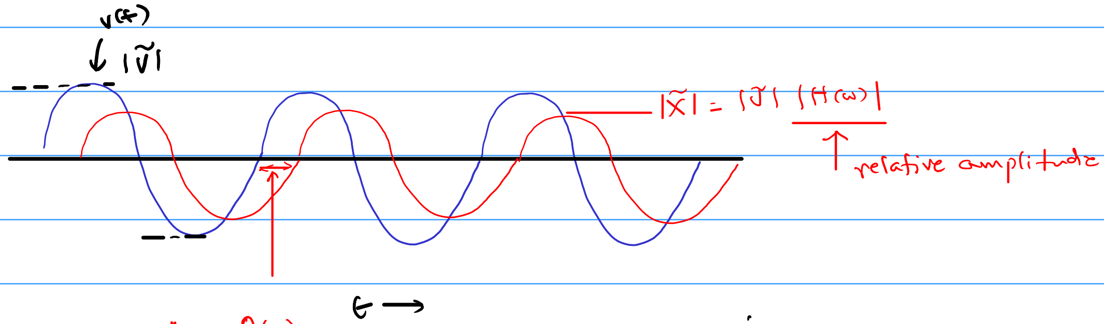
→ express in polar form : $|\tilde{X}|e^{j\theta}$

→ DISCUSSION →

$$|\tilde{X}(\omega)| = \frac{|\tilde{V}|}{\sqrt{1+\omega^2R^2C^2}} , \quad \theta(\omega) = \angle H(\omega) = -\tan^{-1}(\omega RC)$$

$\Rightarrow x(t) = |\tilde{X}(\omega)|\, e^{j(\omega t+\theta)}$ + c.c. term

$$\Rightarrow \boxed{x(t) = 2|\tilde{X}(\omega)| \cos(\omega t+\theta)} \longrightarrow$$

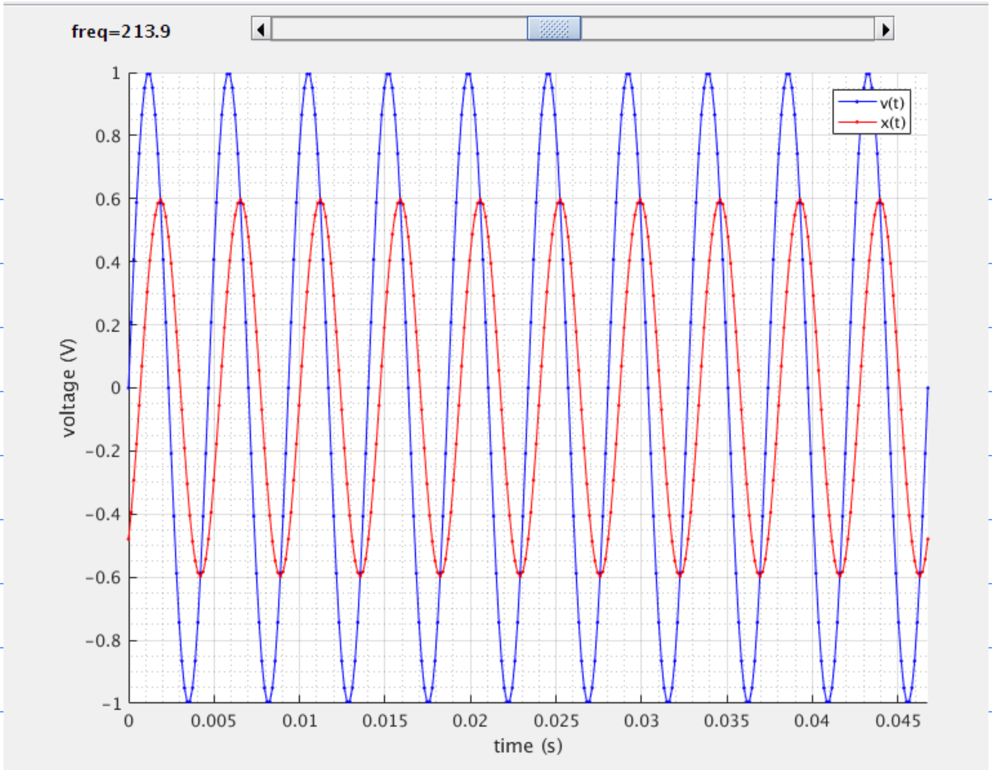


$\Delta = \dfrac{\theta(\omega)}{\omega}$ → why? $e^{j(\omega t+\theta)} = e^{j\omega(t+\underline{\theta/\omega})}$

↑ time-shift.

→ OSCILLOSCOPE DEMO (NUMERICAL)

→ OSCILLOSCOPE DEMO



→ SUMMARY: WHAT HAVE WE DONE TODAY

→ DEMO (AUDIO): MANY SINUSOIDS AT DIFFERENT FREQS = VOICE/MUSIC/ETC.

→ SINUSOIDS CAN BE WRITTEN USING COMPLEX NUMBERS + C.C.

→ PHASOR: coeff. of $e^{j\omega t}$

→ IN "PHASOR DOMAIN": CAP LOOKS LIKE RESISTOR

↳ IMPEDANCE depends on angular freq ω

→ CIRCUITS CAN BE SOLVED IN PHASOR DOMAN $= 2\pi f$

↳ BIG WIN OVER SOLVING DIFF. EQNS.

↳ BUT VALID ONLY IF ALL WAVEFORMS SINUSOIDAL.

→ IN PHASOR DOMAIN: RATIOS OF VOLTAGES/CURRENTS ARE SIMPLE EXPRESSIONS IN ω (eg, $\frac{1}{1+j\omega RC}$)

→ TRANSFER FUNCTIONS H(ω): output = H(ω) × input

"FREQ. DOMAIN" PHASORS

→ CAN EASILY RECOVER TIME DOMAIN (eg x(t), i(t)) From PHASORS.